Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 24, 2002*
Revised: *August 28, 2002*
Accepted: *August 28, 2002*

# Is there really a de Sitter/CFT duality

**Lisa Dyson,[*] James Lindesay[†] and Leonard Susskind[‡]**

*Stellenbosch Institute for Advanced Study*
*Stellenbosch, South Africa*
*E-mail:* ldyson@mit.edu, JLslac@SLAC.Stanford.edu, susskind@sewerrat.stanford.edu

Abstract: In this paper a de Sitter Space version of Black Hole Complementarity is formulated which states that an observer in de Sitter Space describes the surrounding space as a sealed finite temperature cavity bounded by a horizon which allows no loss of information. We then discuss the implications of this for the existence of boundary correlators in the hypothesized dS/cft correspondence. We find that dS complementarity precludes the existence of the appropriate limits. We find that the limits exist only in approximations in which the entropy of the de Sitter Space is infinite. The reason that the correlators exist in quantum field theory in the de Sitter Space background is traced to the fact that horizon entropy is infinite in QFT.

Keywords: AdS-CFT and dS-CFT Correspondence, Models of Quantum Gravity, Thermal Field Theory, Random Systems.



[*]Permanent address, Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

[†]Permanent address, Computational Physics Laboratory, Howard University, Washington, DC 20059, USA.

[‡]Permanent address, Department of Physics, Stanford University, Stanford, CA 94305-4060, USA.

## Contents



## 1. The complementarity principle

Recent evidence suggests that we may live in a space-time that will asymptotically tend to de Sitter Space. If this is so, it is important to understand how quantum gravity should be formulated in such a geometry. Since de Sitter Space has an event horizon many of the questions that confused theorists about the quantum theory of black holes become relevant to cosmology. Perhaps the most important lesson we have learned from black hole quantum mechanics concerns the complementary way that different observers describe events in the black hole environment. That together with the Holographic principle, the UV/IR connection and the counting of black hole microstates is providing a new paradigm for the quantum mechanics of horizons. It is therefore natural to try to apply them to de Sitter Space.

According to the Principle of Black Hole Complementarity [1] the horizon of a black hole may be regarded by an external observer as an impenetrable thermal membrane which can absorb, thermalize and reemit all information. The principle also says that a freely falling observer encounters nothing special at the horizon. The principle has recieved strong support from the study of gravity in AdS space and its equivalence to the boundary conformal field theory. The horizon of a de Sitter Space is structurally very similar to that of a black hole. A static patch of de Sitter Space is described by the metric

$$ds^2 = R^2\left[\left(1-r^2\right)dt^2 - \left(1-r^2\right)^{-1}dr^2 - r^2d^2\Omega\right]. \tag{1.1}$$

In this case the horizon surrounds the observer.

It is well known that the static patch has an entropy given by

$$S = \frac{Area}{4G} = \frac{4\pi R^2}{4G}. \tag{1.2}$$

The proper temperature at $r = 0$ is given by $T = 1/2\pi R$. More generally the local proper temperature is given by

$$T(r) = \frac{1}{2\pi R\sqrt{1-r^2}}. \tag{1.3}$$

Note that as in the black hole case the local proper temperature formally diverges near the horizon.



JHEP08(2002)045

The analog of the Black Hole Complementarity Principle for de Sitter space can be expressed as follows: An observer in de Sitter Space sees the surrounding spacetime as a finite closed cavity bounded by a horizon. The cavity is described by a thermal ensemble at coordinate temperature $1/2\pi$. As for any closed system information can never be lost but can be scrambled or thermalized at the hot horizon. Also as in the black hole case, a freely falling observer will experience nothing out of the ordinary when passing through the horizon. These complementary descriptions of the horizon comprise the Complementarity Principle for de Sitter Space.

Next let us consider the attempt by Strominger [2] to formulate a holographic description of de Sitter Space. The proposal is inspired by the AdS/CFT duality and is based on the fact that the symmetries of de Sitter Space act on the asymptotic boundary of de Sitter Space as conformal mappings. In particular time-translation in the static patch is a dilatation that preserves a point $p$ on the boundary. Ordinarily this dilatation invariance would allow us to construct a local field at $p$. Take any operator in the static patch at time $t$. This operator does not have to be a local bulk operator. Now use time translation to translate it back in time toward $t = -\infty$. From the boundary point of view this is a dilation which should shrink the support of the operator to a point. One of the key assumptions of the dS/CFT correspondence is that the matrix elements of such an operator behave like

$$\exp \gamma t \tag{1.4}$$

where the real part of $\gamma$ is positive. Factoring off this exponential leaves a set of matrix elements that are assumed to define local field operators in the boundary CFT. In what follows it will be shown that the assumption of eq. (1.4) must be wrong and that no local field is defined by the above procedure.

## 2. Correlations in QFT

As an example we will follow Bousso, Maloney and Strominger who consider a massive scalar field $\phi$ in $2 + 1$ dimensional de Sitter Space. Straightforward study of the wave equation shows that correlation functions behave like eq. (1.4) with

$$\gamma = H \pm i\sqrt{m^2 - H^2} \tag{2.1}$$

where $H$ is the Hubble parameter proportional to $1/R$. As a particular example consider the two-point function $\langle\phi(t)\phi(t')\rangle$ where both points are evaluated at the spatial point $r = 0$. Equation (1.4) together with time translation invariance in the static patch imply that the correlation function behaves like

$$\langle\phi(t)\phi(t')\rangle \sim \exp -\gamma|t - t'|\,. \tag{2.2}$$

Evidently the correlator exponentially tends to zero with large time separation in the static patch.



JHEP08(2002)045

## 3. Correlations for systems of finite entropy

The exponential decay of correlations has an interesting explanation from the point of view of the thermal cavity picture. It's a general property of correlation functions in thermal equilibrium that they exponentially die off with large time. Typically the coefficient in the exponential is some dissipation coefficient. The de Sitter Space correlation functions are simply reflecting the phenomena of dissipation in a thermal bath. However, as we shall see, this is only true if the thermal bath has infinite entropy. For example this would be the case for an infinite heat bath. For a closed system of finite entropy the behavior is much more complicated at asymptotic times. The essential point is that any quantum system with finite thermal entropy must have a discrete spectrum. This is because the entropy is essentially the logarithm of the number of states per unit energy. Of course in many circumstances including classical physics, the entropy arises from an integral over a continuum of states. However it is generally understood that the true information theoretic entropy of such systems contains an additive infinity. Thus if we believe that the entropy is finite and that it has the usual statistical meaning, the spectrum must be discrete. Let us now consider a general finite closed system described by a thermal density matrix and a thermal correlator of the form

$$F(t) = \langle A(0)A(t)\rangle = \frac{1}{Z} Tr e^{-\beta H} A(0) e^{iHt} A(0) e^{-iHt} . \tag{3.1}$$

By finite we simply mean that the spectrum is discrete and the entropy finite. Inserting a complete set of (discrete) energy eigenstates gives

$$F(t) = \frac{1}{Z}\sum_{ij} e^{-\beta E_i} e^{i(E_j - E_i)t} |A_{ij}|^2 . \tag{3.2}$$

For simplicity we will assume that the operator $A$ has no matrix elements connecting states of equal energy. This means that the time average of $F$ vanishes.

Let us now consider the long time average of $F(t)F^*(t)$.

$$L = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} dt F(t) F^*(t) \tag{3.3}$$

Using eq. (3.2) it is easy to show that the long time average is

$$L = \frac{1}{Z^2}\sum_{ijkl} e^{-\beta(E_i+E_k)} |A_{ij}|^2 |A_{kl}|^2 \delta_{(E_j - E_l + E_k - E_i)} . \tag{3.4}$$

where the delta function is defined to be zero if the argument is non-zero and 1 if it is zero. The long time average $L$ is obviously non-zero and positive. Thus it is not possible for the correlator $F(t)$ to tend to zero as the time tends to infinity and the limits required by the dS/CFT correspondence can not exist. The value of the long time average for such finite systems can be estimated and it is typically of the order of some power of $e^{-S}$ where $S$ is the entropy of the system. This observation allows us to understand why it tends to zero in the (bulk) QFT approximation. In studying QFT in the vicinity of a horizon it is well



JHEP08(2002)045

known that the entropy is UV divergent. This is due to the enormous number of short wave length modes near the horizon. Any phenomenon which crucially depends on the finiteness of horizon entropy will be gotten wrong by the approximation of QFT in a fixed background. How exactly do the correlations behave in the long time limit? The most probable answer is not that they uniformly approach constants given by the long time averages. The expected behavior is that they fluctuate chaotically. A large fluctuation which reduces the entropy by amount $\Delta S$ has probability $e^{-\Delta S}$. Thus we can expect large fluctuations in the correlators at intervals of order $e^{S}$. These fluctuations are closely related to the classical phenomenon of Poincare recurrences. In the appendix to this paper a numerical study of such correlations is presented for finite systems with random matrix hamiltonians. It is found that the large time behavior of correlators is chaotic "noise" with the long time average given by

$$L \sim e^{-S} \,. \tag{3.5}$$

Does this mean that an exact dS/CFT correspondence can not exist? This is not entirely clear. Let's suppose that the CFT is something like a euclidean version of a gauge theory and that the correspondence is similar to the AdS/CFT case. Quantities associated with the bulk would be described by nonlocal objects such as Wilson loops of finite size; the larger the Wilson loop the further the corresponding point from the past boundary. A correlator at two times would be related to the product of concentric Wilson loops. Now in the usual case the local gauge invariant operators such as the energy momentum tensor can be obtained in terms of shrinking Wilson loops. The argument given in this paper is that the limit in which one of the loops shrinks to zero can not exist but instead must fluctuate in endless Poincare recurrences. Can a gauge theory behave this way and can it form a representation of the conformal group? Perhaps but it would certainly be a very unfamiliar kind of theory. Another point concerns the implications for string theory in de Sitter Space. If boundary correlators made any sense they would be the natural candidates for computables in string theory much in the same way that string theory defines observables in AdS. The non-existence of these correlators raises the question of what objects string theory can define in de Sitter Space [3]. At the moment there is no answer to this question. Finally we should point out that the magnitude of the chaotic effects that we have discussed is of order $e^{-S} = e^{-A/4G}$ where $A$ is the horizon area of de Sitter Space. This has the form of a non-perturbative effect in the gravitational coupling. Maldacena [4] has discussed very similar effects for eternal black holes in AdS and has suggested that they arise from non-perturbative sums over space-time topology. This may also be possible in de Sitter Space.

## Acknowledgments

The authors would like to thank Bernard Lategan and Hendrik Geyer for their kind hospitality at Stellenbosch University and the Stellenbosch Institute for Advanced Study in South Africa where this work was done.

## A. An example calculation for a system of finite entropy

As an illustration of the late times behavior of systems with finite entropy, we will construct a model and numerically explore its behavior at large times. We will study the thermal correlators of operators A given by the functional form of a scalar field $\phi$ that would behave for large times as in equations (1.4) and (2.1) if the energy spectrum were continuous. The forms $A_{ij}$ are then directly related to a Fourier transform of this field, with the energy differences $E_i - E_j$ as its argument, and $\gamma$ as a parameter.

We model the finite entropy system by constructing a finite square matrix of random elements with a normal (gaussian) distribution and using the eigenvalues of this matrix as the energy levels of the thermal system. The standard deviation of the gaussian distribution reflects in the density of the spacing of these energy levels. We will assume that the thermal bath has entropy given by $S = -Tr\rho log\rho$ for the thermal density matrix $\rho$. The entropy is varied by varying the temperature. The time dependence of the absolute value of the correlator as given in equation (3.2) can then be examined.

As the entropy increases, the amplitude of significant recurrences within a given time frame was seen to decrease, as illustrated in the first set of diagrams (figures 1 $a$, $b$, and $c$) for matrix sizes $N = 5$, 10, and 20 respectively.

The time scale for recurrence of significant fluctuations above a fixed absolute magnitude was seen to increase roughly proportionally to $\mathrm{e}^S$. This is demonstrated in the next set of graphs (figures 2 and 3).

These graphs for matrix size $N = 20$ demonstrate that fluctuations of order 0.4 are seen to occur on time scales of roughly $10^4$ units, fluctuations of order 0.5 on time scales of $10^6$, and fluctuations of order 0.7 on time scales of $10^8$. Such fixed order fluctuations are found to have recurrence times that increase very rapidly with the size of the fluctuation. The next graph (figure 4) demonstrates the recurrence time for significant fixed size fluctuations as a function of the size of the fluctuation for a system with $N = 10$ levels.

The numerical dependence of these late time fluctuations upon the various time scales was explored in an attempt to determine the scaling variables. Assuming that all of the external parameters (such as de Sitter scale H and scalar mass m) are held fixed, there are several time scale parameters in the model which might be relevant to the recurrences; for example, the entropy timescale $\mathrm{e}^S$, the inverse range in energy eigenvalues defined for the thermal system $1/\Delta E$, and the inverse of the average energy spacing between the levels $\mathrm{N}/\Delta E$, along with various combinations of these time scales. The functional behavior found is illustrated in figure 5.

As a final consideration, the long time average L from equation (3.4). The functional dependency of L is displayed in the final diagram (figure 6), which shows the logarithm of the long time average as a function of the entropy for a particular set of random matrices. This numerical result is consistent with the form $L \sim e^{-S}$, as previously stated.

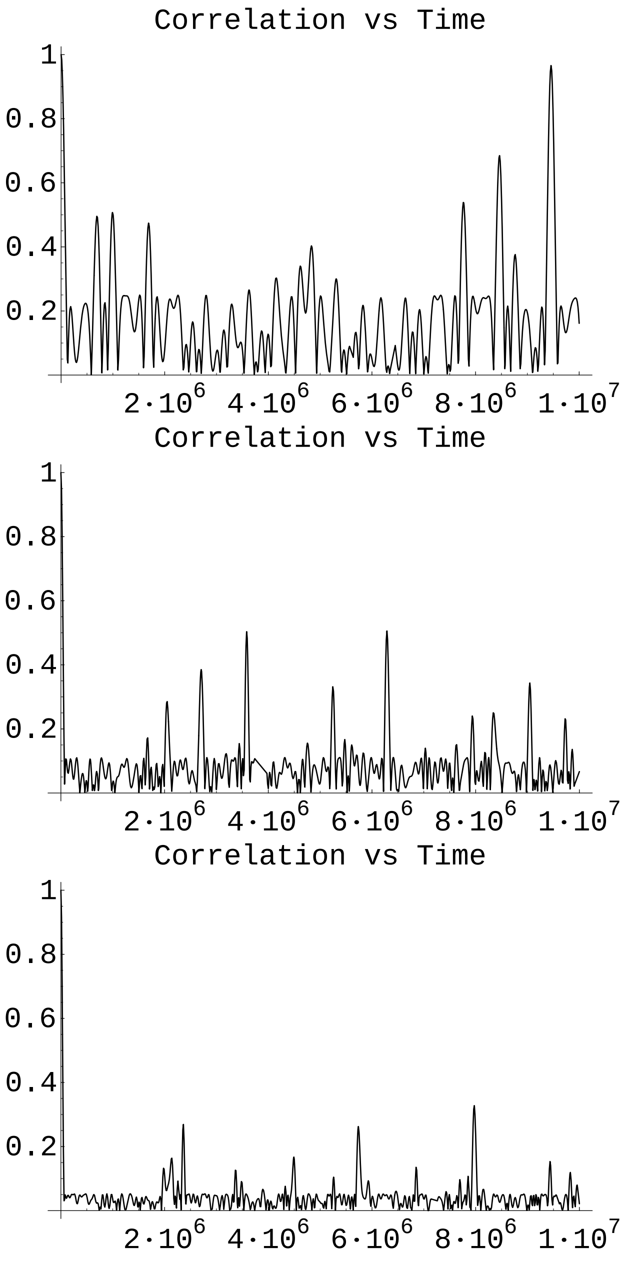


**Figure 1:** Large time behavior of correlation functions for $(a)$ $N = 5$, $(b)$ $N = 10$ and $(c)$ $N = 20$.

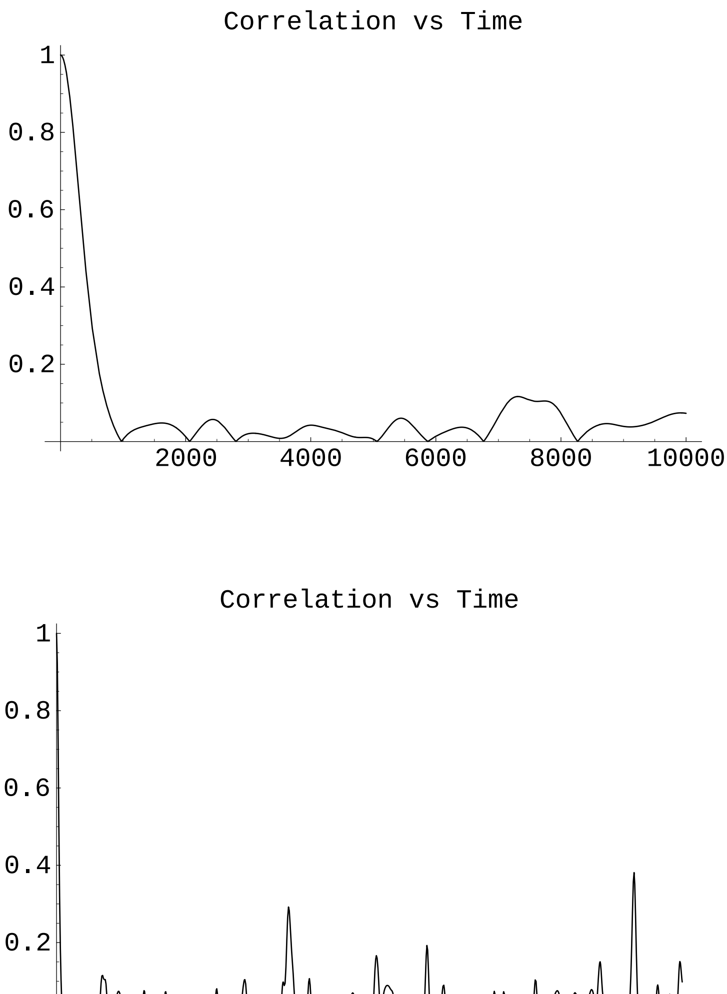


**Figure 2:** Correlation recurrences for $N = 20$, showing initial falloff and fluctuations.

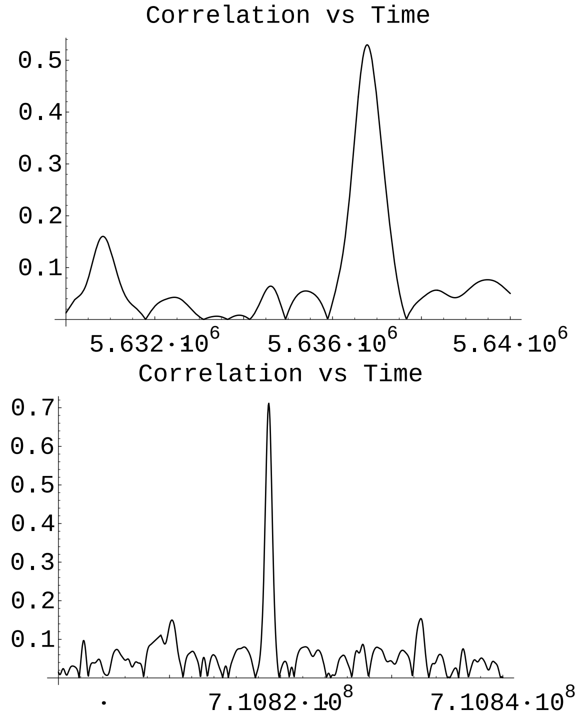


**Figure 3:** Correlation recurrences for $N = 20$: $(a)$ 50%, $(b)$ 70%.

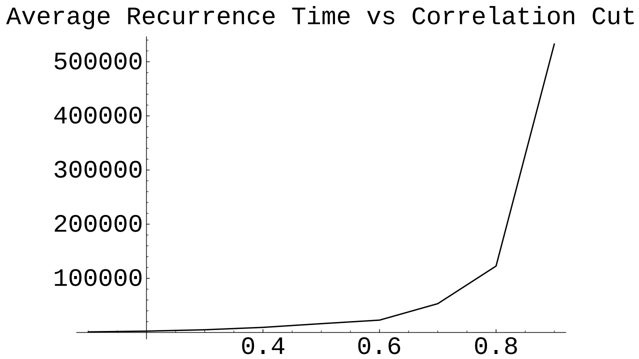


**Figure 4:** Recurrence time as a function of absolute size of fluctuations.

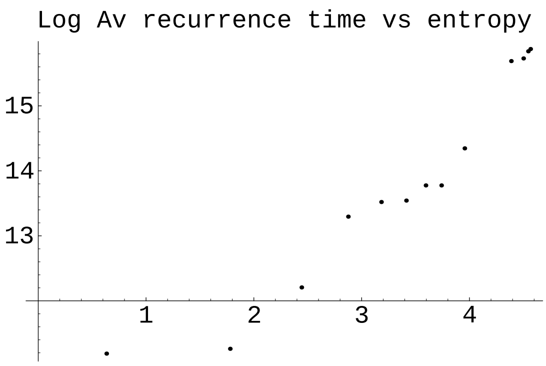


**Figure 5:** Recurrence time as a power of $e^S$.

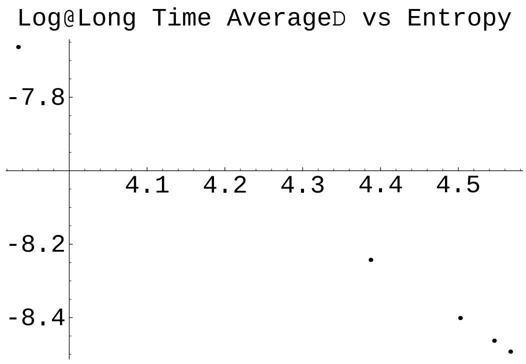


**Figure 6:** Behavior of the log of the long time average as a function of the entropy.

## References


[1] L. Susskind, L. Thorlacius and J. Uglum, *The stretched horizon and black hole complementarity*, *Phys. Rev.* **D 48** (1993) 3743 [hep-th/9306069];
C.R. Stephens, G. 't Hooft and B.F. Whiting, *Black hole evaporation without information loss*, *Class. and Quant. Grav.* **11** (1994) 621 [gr-qc/9310006];
complementarity in the context of de Sitter space is also discussed in T. Banks and W. Fischler, *M-theory observables for cosmological space-times*, hep-th/0102077.

[2] R. Bousso, A. Maloney and A. Strominger, *Conformal vacua and entropy in de Sitter space*, *Phys. Rev.* **D 65** (2002) 104039 [hep-th/0112218] references therein.

[3] E. Witten, *Quantum gravity in de Sitter space*, hep-th/0106109.

[4] J.M. Maldacena, *Eternal black holes in anti-de Sitter*, hep-th/0106112.